In-Flight Estimation of the Cassini Spacecraft's Inertia Tensor

Allan Y. Lee*
Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California 91109
and
Julie A. Wertz†
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

Nomenclature

$\boldsymbol{H}_{\text{RWA}}$ = angular momentum vector of the three reaction wheels, N-m-s
$\boldsymbol{H}_{\text{SC}}$ = angular momentum vector of the spacecraft, N-m-s
I_{RWA} = 3×3 diagonal matrix with the moments of inertia of three reaction wheels on its main diagonal, kg-m^2
I_{SC} = 3×3 inertia matrix of the spacecraft, kg-m^2
q_i = Euler parameters ($i = 1, 2, 3, 4$)
T = 3×3 coordinate transformation matrix from the reaction wheel frame to the spacecraft coordinate frame
$\boldsymbol{\rho}$ = reaction wheel's angular rate vector, rad/s
ρ_i = angular rate of the ith reaction wheel about its spin axis, rad/s
$\boldsymbol{\Omega}$ = spacecraft's angular velocity vector, rad/s
ω_i = ith component of the spacecraft's angular velocity vector ($i = x, y, z$ axes), rad/s

Introduction

THE Cassini spacecraft was launched on 15 October 1997. After an interplanetary cruise of almost seven years, the Cassini spacecraft will arrive at Saturn in July 2004. Major science objectives of the Cassini mission include investigations of the configuration and dynamics of Saturn's magnetosphere, the structure and composition of the rings, the characterization of several of Saturn's icy moons, and others. Detailed descriptions of various science instruments carried onboard the Cassini spacecraft are given in Ref. 1.

Several attitude-control algorithms onboard the spacecraft use knowledge of the Cassini spacecraft 3×3 inertia matrix. This matrix is used in both the attitude-control fault protection algorithms and attitude estimator. Thruster vector control algorithms, which are employed to control the spacecraft attitude during all propulsive maneuvers, also make use of the onboard knowledge of the inertia matrix. Knowledge of the inertia matrix is also used by the reaction wheel actuator (RWA) controller, which is used to maintain precision spacecraft attitude control during imaging of science targets. As such, a highly accurate estimate of this inertia matrix is important to spacecraft operations. An overview of the Cassini attitude control subsystem design is given in Ref. 2.

Before launch, Cassini's inertia matrix was estimated by adding together the moments of inertia of the individual components of the spacecraft. The moments of inertia of individual components were computed with respect to the predicted center of mass of the overall spacecraft before being summed. After launch the onboard spacecraft inertia matrix is updated periodically using estimates of how much propellant has been used to date, as well as any discrete events (for example, the deployment of the magnetometer boom) that would affect the inertia matrix. The inertia matrix of the spacecraft on 15 March 2000, using the "sum-of-all-components" method, is estimated to be

$$I_{\text{SC}} = \begin{bmatrix} 8810.8 & -136.8 & 115.3 \\ -136.8 & 8157.3 & 156.4 \\ 115.3 & 156.4 & 4721.8 \end{bmatrix} \quad \text{kg-m}^2 \tag{1}$$

This method of calculating the inertia tensor had not been validated in-flight using an independent approach until this study. The goal of this Note is to propose and validate a methodology that can be used to estimate the Cassini spacecraft inertia tensor.

In the literature Tanygin and Willians[3] showed how to estimate the spacecraft inertia matrix of a spinning rigid body using Euler's equation. The Euler equation approach leads to a formulation that must account for energy flow into and out of the spacecraft. Any unknown dissipative torque within the spacecraft is dealt with as measurement noise with an estimated covariance matrix. By contrast, Peck[4] proposed a method that is based on the conservation of the total angular momentum (instead of energy) of the spacecraft. This approach eliminates the need to estimate the uncertainty associated with the energy dissipation process. The conservation of angular momentum approach is independently proposed in this study to estimate the inertia matrix of the Cassini spacecraft. The usefulness of this approach is established via its application on the Cassini spacecraft.

Problem Formulation

When a spacecraft is slewed using the RWAs, the total angular momentum of the spacecraft expressed in an inertial coordinate frame is conserved. This conservation occurs because the addition of angular momentum on the spacecraft caused by external torque, such as solar radiation torque, is typically very small over the duration of the slew. Approximate magnitudes of the external torque experienced by the Cassini spacecraft are given in Ref. 5. On 15 March 2000 the largest per-axis external torque from all sources was about the spacecraft's X axis and was less than 1.5×10^{-5} N-m. The conservation of angular momentum allows the total angular momentum evaluated just prior to the beginning of the slew to be set equal to the total angular momentum evaluated throughout the slew. This equality gives an equation for each sample time step throughout the slew with only one unknown I_{sc}, which can then be estimated via a least-squares approach. Note that I_{sc} contains the moments of inertia of the three stationary reaction wheels.

Over a spacecraft slew good estimates of the following quantities are available, either from direct measurement prior to launch or from the telemetry data sent down from the spacecraft: 1) spacecraft angular rates (ω_x, ω_y, and ω_z), 2) spin rates with respect to the spacecraft of the three RWAs (ρ_1, ρ_2, and ρ_3), 3) spacecraft Euler parameters (q_1, q_2, q_3, and q_4), 4) inertia matrix of the three RWAs (I_{RWA}), and 5) transformation matrix from the three RWA spin axes to the XYZ body coordinate frame T.

The total angular momentum vector of the spacecraft, as expressed in the spacecraft body frame, has two components: $\boldsymbol{H}_{\text{Total}} = \boldsymbol{H}_{\text{SC}} + \boldsymbol{H}_{\text{RWA}}$. The component caused by the spacecraft rates is $\boldsymbol{H}_{\text{SC}} = I_{\text{SC}}\boldsymbol{\Omega}$, where $\boldsymbol{\Omega} = [\omega_x, \omega_y, \omega_z]^T$. To determine the angular momentum of the RWAs, first define $\boldsymbol{\rho} = [\rho_1, \rho_2, \rho_3]^T$. The RWA spin axes are oriented 120 deg apart when they are projected on the spacecraft X–Y plane. All three spin axes are 54.73 deg off the spacecraft's $+Z$ axis. Knowing the orientations of the three RWAs relative to the XYZ body frame, a transformation matrix T (from the RWA coordinate frame to the spacecraft XYZ body coordinate frame) can be determined.

To find $\boldsymbol{H}_{\text{RWA}}$ in body coordinates, simply multiply $\boldsymbol{\rho}$ by the inertia matrix for the RWAs, and then multiply by the transformation matrix T. The component of $\boldsymbol{H}_{\text{RWA}}$ caused by spacecraft rates has already been accounted for in $\boldsymbol{H}_{\text{SC}}$:

$$\boldsymbol{H}_{\text{RWA}} = T I_{\text{RWA}} \boldsymbol{\rho} \tag{2}$$

Received 26 March 2001; revision received 3 October 2001; accepted for publication 10 October 2001.

*Cassini Attitude Control Element Manager, Mail Stop 230-104, 4800 Oak Grove Drive; Allan.Y.Lee@jpl.nasa.gov.

†Graduate Research Assistant, Space Systems Laboratory, 77 Massachusetts Avenue, Room 37-340; jwertz@mit.edu.

The conservation of angular momentum is only valid in an inertial coordinate system. As such, a transformation matrix P, defined here from the J_{2000} inertial frame to the body coordinate frame, must be defined. The transformation matrix is computed using the four Euler parameters (q_1, q_2, q_3, and q_4). Multiplying the total angular momentum of the spacecraft in body coordinates by the inverse of the transformation matrix P gives the total angular momentum vector in the inertial coordinate frame. The resultant vector is approximately conserved over a spacecraft slew:

$$\boldsymbol{H}_{\text{Total}} = P^{-1} I_{\text{SC}} \boldsymbol{\Omega} + P^{-1} T I_{\text{RWA}} \boldsymbol{\rho} \tag{3}$$

The spacecraft is quiescent just prior to the slew, with all angular rates approximately zero. As such, the initial angular momentum vector is given by

$$\boldsymbol{H}_{\text{Total}}(0) = P^{-1}(0) T I_{\text{RWA}} \boldsymbol{\rho}(0) \tag{4}$$

Invoking the conservation of angular momentum, one gets

$$P(t)^{-1} I_{\text{SC}} \boldsymbol{\Omega}(t) + P(t)^{-1} T I_{\text{RWA}} \boldsymbol{\rho}(t) \approx P^{-1}(0) T I_{\text{RWA}} \boldsymbol{\rho}(0) \tag{5}$$

Now, for the sake of simplicity, consider the special case in which the spacecraft slews about one axis at a time. In this case the rate components about the other two axes go to zero. For example, for a slew about the X axis, Eq. (5) becomes

$$I_{\text{SC}} \begin{bmatrix} \omega_x(t) \\ 0 \\ 0 \end{bmatrix} = P(t) P^{-1}(0) T I_{\text{RWA}} \boldsymbol{\rho}(0) - T I_{\text{RWA}} \boldsymbol{\rho}(t) \tag{6}$$

Denote the right-hand side of Eq. (6) by a new vector: $\boldsymbol{Q}(t) = [Q_x(t) \;\; Q_y(t) \;\; Q_z(t)]^T$. Using this notation, the first component of the vector matrix Eq. (6) is $I_{xx}\omega_x(t) = Q_x(t)$. In Eq. (6) both $\omega_x(t)$ and $Q_x(t)$ will take on a new value for each sample instant t throughout the slew, producing a separate equation for each sample instant. If $\boldsymbol{\varpi}_x$ and $\boldsymbol{Q}_x$ represent $N_S \times 1$ column vectors of data points from all sample instances (N_S is the total number of samples), a least-squares approach can be used to find the best estimate of I_{xx}:

$$\hat{I}_{xx} = [\boldsymbol{\varpi}_x^T \boldsymbol{\varpi}_x]^{-1} \boldsymbol{\varpi}_x^T \boldsymbol{Q}_x \tag{7}$$

This process can be repeated for I_{yx} and I_{zx} using the pairs of vectors $[\boldsymbol{\varpi}_x, \boldsymbol{Q}_y]$ and $[\boldsymbol{\varpi}_x, \boldsymbol{Q}_z]$, respectively. The entire process can then be repeated for slews about the Y and Z axes as well. This process will give one estimate for each of the moments of inertia (MOI) and two estimates for each one of the products of inertia (POI). The two POI estimates have been averaged together to obtain the best estimate.

An alternative to the approach just described is to estimate all six independent components of the inertia matrix simultaneously using all of the slew data (X-, Y-, and Z-axis slews) at the same time. The axis-by-axis approach just described was used because of its relative simplicity.

Furthermore, the accuracy of the estimation could be improved by one or more of the following approaches: 1) increase the number of samples via lengthening the spacecraft per-axis slew times, and 2) adopt a weighted least-squares approach. In the weighted least-squares approach the quantities given in Eq. (6) are first "normalized" using their respective estimation uncertainties before computing the least-squares estimation via Eq. (7). In this way accurately estimated quantities (with smaller estimation uncertainties) are weighted more heavily in computing $\hat{I}_{\text{SC}}$. Approximate magnitudes of the estimation uncertainties of the spacecraft's per-axis rates and the RWA spin rates are given in the Discussion of Results section.

Input Data

At the time when this study was made, only one maneuver had been done with the Cassini spacecraft using the RWAs. This maneuver was done on 15 March 2000 and lasted four hours. As seen in Fig. 1, this maneuver consisted of a slew about the Y axis, followed by a slew about the X axis, another slew about the Y axis, a slew about the Z axis, and finally a very small slew about the Y axis. Telemetry data for the Euler parameters, spacecraft per-axis rates, and RWA spin rates are available over the entire slew duration, at a sample frequency of 0.25 Hz.

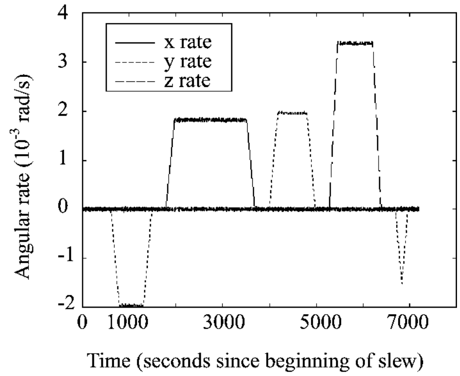

Fig. 1 Time histories of spacecraft per-axis angular rates on 15 March 2000.

Discussion of Results

The data from 15 March 2000 were analyzed using the proposed methodology. The resulting best estimate for the inertia matrix of the spacecraft was

$$\hat{I}_{\text{SC}} = \begin{bmatrix} 8655.2 & -144 & 132.1 \\ -144 & 7922.7 & 192.1 \\ 132.1 & 192.1 & 4586.2 \end{bmatrix} \text{ kg-m}^2 \tag{8}$$

When compared to the inertia matrix obtained using the existing method [see Eq. (1)], this result validates the existing method, as they are reasonably close. The current results are consistently lower than their counterparts given in Eq. (1) by at most 3%. This offset could point to a bias in the estimate of the spacecraft inertia matrix prior to launch. A bias in the prelaunch estimate is possible because the knowledge requirement for the MOI of the "dry" spacecraft is $\pm 10\%$. Also, the POI estimates are within 40 kg-m^2 of their counterparts given in Eq. (1). The magnitudes of the POI estimates are all larger than their counterparts given in Eq. (1), which again could be evidence of a bias. Prelaunch, the knowledge requirement for the POI of the dry spacecraft, is ± 75 kg-m^2.

The estimation uncertainty matrix associated with $\hat{I}_{\text{SC}}$ is derived in Ref. 5. To compute the estimation uncertainty of $\hat{I}_{\text{SC}}$, note that the one-sigma estimation uncertainty of the per-axis spacecraft rates is 1×10^{-5} rad/s. Also, the one-sigma estimation uncertainty of the RWA spin rates is 0.42 rad/s. Three quantities, which have been measured/estimated with great accuracy, were not included in the uncertainty analysis. The quantities are 1) RWA inertia property, known to better than 0.5%; 2) RWA spin axes relative to the spacecraft coordinate frame, known to better than 0.5 deg; and 3) the measurement uncertainty of the spacecraft's attitude relative to inertial frame is estimated to better than 5×10^{-6} rad. The computed one-sigma estimation uncertainty matrix of $\hat{I}_{\text{SC}}$ is

$$\sigma_{\hat{I}_{\text{SC}}} = \begin{bmatrix} 5.3 & 2.4 & 1.5 \\ 2.4 & 11.5 & 1.8 \\ 1.5 & 1.8 & 2.0 \end{bmatrix} \text{ kg-m}^2 \tag{9}$$

Conclusions

The least-squares estimate of the Cassini spacecraft's inertia matrix obtained through the conservation of angular momentum method just described agrees closely with that determined by the existing method. This agreement validates the conservation of angular momentum method. Using this method, the moments and products of inertia of a spacecraft could be easily estimated whenever telemetry data associated with slewing the spacecraft by the reaction wheels are available.

Acknowledgments

The research described in this Note was carried out by the Jet Propulsion Laboratory (JPL), California Institute of Technology, under a contract with NASA. The authors wish to thank Kevin Barltrop, William Breckenridge, Larry Chang, Gene Hanover, Randy Lin, Glenn Macala, and David Skulsky, their colleagues at JPL, for many helpful discussions. The second author would also like to thank David Miller of the Massachusetts Institute of Technology Space Systems Laboratory for all of his support. Also, we would like to thank the reviewers and the Associate Editor of the *Journal of Spacecraft and Rockets* for their review of an earlier version of this Note.

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C. A. Kluever
Associate Editor

Impingement Angle Dependence of Erosion Rate of Polyimide in Atomic Oxygen Exposures

Kumiko Yokota,* Masahito Tagawa,† and Nobuo Ohmae‡
Kobe University, Kobe 657-8501, Japan

Introduction

It has been recognized that atomic oxygen is one of the most important factors that influences the erosion of many space materials, especially polymeric materials, in low Earth orbit. Although a number of polymeric materials are utilized in space systems, polyimide is one of the most widely used polymeric materials in spacecraft applications. The erosion rate of polyimide film due to atomic oxygen attack in low Earth orbit was reported to be 3.00×10^{-24} cm^3/atom, and, hence, polyimide film has been used as one of the reference materials to evaluate the erosion rate of other materials.[1] For the reference material, erosion rates at various exposure conditions need to be well understood. However, the existing basic knowledge on the erosion of polyimide due to atomic oxygen is not extensive enough for predicting the erosion of polyimide film under various exposure conditions. One of the major factors that influences the erosion rate of polyimide film is the impingement angle of atomic oxygen. However, an accurate measurement of the impingement angle dependence has not been reported.

In this Note, we report ground-based experimental results of the impingement angle dependence of the erosion rate of polyimide film due to exposure to a hyperthermal atomic oxygen beam. In situ mass loss measurements were made during the atomic oxygen exposure by using a quartz crystal microbalance (QCM), so that any possible disturbance influencing the reliability of the postprocess erosion measurement such as moisture absorption, contamination, or unexpected change in the beam conditions during the exposure could be eliminated.

Experiments

The laser detonation-type atomic oxygen beam source, which was originally invented by Physical Sciences, Inc. (PSI), was used in this study.[2] Details of the experimental apparatus are reported elsewhere.[3] The translational energy of the atomic oxygen beam used in this study was approximately 4.6 eV, whereas the beam flux at the sample position was measured at 3.0×10^{14} atom/cm^2/s by using a silver-coated QCM.[4] The polyimide film used in this study was the pyromelliticanhydride(PMDA)-oxydianiline (ODA) polyimide supplied by Toray Industries, Inc. (Semicofine SP-510). The polyimide film was spin coated on the QCM sensor crystal and annealed at 150°C and then at 300°C. Details of the sample preparation are reported in Ref. 5. The polyimide film, thus prepared, was examined by x-ray photoelectron spectroscopy, and it was confirmed that the surface structure was similar to that of Kapton-H®, which is a commercially available polyimide film. The erosion rate of the polyimide film was calculated from the change in the resonant frequency of the QCM during the atomic oxygen beam exposure. The frequency of the QCM was measured every 10 s with a frequency resolution of 0.1 Hz, which corresponds to a mass resolution of 2 ng. The temperature of the film was controlled with an accuracy of 0.1°C. Before the mass loss measurements, the polyimide film was exposed to atomic oxygen (6×10^{17} atoms/cm^2) to saturate the surface oxygen content of the sample. This is done to avoid the effect of nonlinear mass loss phenomenon at the beginning of atomic oxygen exposure to pristine polyimide surfaces.[6]

Results and Discussion

Figure 1 displays the frequency shift of the QCM during atomic oxygen beam exposures at impingement angles from 0 to 90 deg. The impingement angle was taken with respect to the surface normal. A good linear relationship between the frequency shift and exposure time, that is mass loss and atomic oxygen fluence, was observed at all impingement angles. The good linearity of the mass loss with fluence was also identified for larger timescales.[6] The results shown in Fig. 1 were obtained at a sample temperature of 38°C, but similar results were also observed at sample temperatures from 15 to 70°C. The slope of the mass loss rate at every impingement angle was calculated by a least-squares fit and was plotted against the impingement angle. The results are presented in Fig. 2. It is clear that the rate of frequency shift, or erosion rate, of polyimide depends on the impingement angle and that the dependence obeys a cosine law as indicated by the solid line in Fig. 2. Note that the

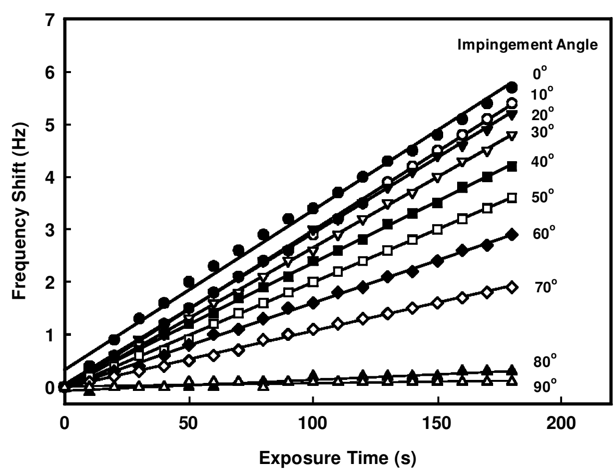

Fig. 1 Resonant frequency shift of polyimide-coated QCM under the atomic oxygen exposures at impingement angles from 0 to 90 deg.

Received 20 April 2001; revision received 2 July 2001; accepted for publication 6 July 2001.

*Research Staff, Department of Mechanical Engineering, Faculty of Engineering, 1-1 Rokko-dai, Nada; yokota@mech.kobe-u.ac.jp.

†Associate Professor, Department of Mechanical Engineering, Faculty of Engineering, 1-1 Rokko-dai, Nada. Member AIAA.

‡Professor, Department of Mechanical Engineering, Faculty of Engineering, 1-1 Rokko-dai, Nada.